

# Bottom trapped internal waves over the Malin Sea continental slope

Nataliya Stashchuk<sup>a</sup>, Vasiliy Vlasenko<sup>a</sup>,

<sup>a</sup>*School of Marine Science and Engineering, University of Plymouth, Drake Circus,  
Plymouth, PL4 8AA, UK*

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## Abstract

Three-dimensional dynamics of internal tides on the continental slope of the Malin Sea is studied numerically and validated with the observational data collected on the 88-th cruise of the RRS “James Cook” in June-July 2013. Observed in-situ bottom intensification of baroclinic tidal currents was reproduced in a series of numerical experiments using the Massachusetts Institute of Technology general circulation model. The baroclinic tidal dynamics is explained in terms of superposition of two wave processes developing in the area. One of them is a tidal beam generated at the shelf break and radiating tidal energy downward into abyss. The second important process identified here was a bottom trapped internal wave, generated by the tidal flow over a local prominence. This internal wave was trapped by the topography at the depths below 800 m and propagated northward along the slope.

*Key words:* Baroclinic tides, bottom trapped internal waves, the Malin Sea

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## 1. Introduction

The Malin Sea is a marginal sea bounding the lowlands of Scotland and Ireland. It covers the area restricted by  $57^{\circ}\text{N}$  latitude at the north and  $10^{\circ}\text{W}$  longitude at the west. The main part of the Malin Sea is shallow with the depths typically ranging between 150 and 200 m. Offshore the continental slope is steep at the south (supercritical to internal  $M_2$  tide) and has a number of small-scale canyons and a large-scale indentations between  $55.5^{\circ}\text{N}$  and  $55.8^{\circ}\text{N}$ , Figure 1.

The spatial variability of the bottom topography along the slope suggests existence of a variety of three-dimensional dynamical processes developing in the area that can take the form of barotropic and baroclinic tides, short-scale internal waves, quasi-stationary currents and mesoscale eddies. Historically, most of previous in-situ observations were focussed on the analysis of cross shelf/slope variability of internal tides and tidally generated internal waves (Sherwin (1988); Small *et al.* (1999a); Hallock *et al.* (2000); Small (2003)) and their effect on the marine environment (Inall *et al.*, 2000, 2001; Rippeth and Inall, 2002). Even though a three-dimensional nature of generated wave was obvious in the results of the SESAME experiment (Small *et al.*, 1999a), specifically, in SAR images of recorded internal wave packets, the interpretation of the observational data was conducted based basically either on a two-dimensional theory (Stashchuk and Vlasenko (2005)), or in terms of the mechanism of refraction of internal waves (Small (2000, 2003)). The three-dimensional structure of observed internal waves was not the focus of these studies. It is worth mentioning that the experimental studies in the area mostly ignored an along slope variability, although the paper by Souza

26 *et al.* (2001) investigated a water transport along the Malin shelf/slope, and  
27 the JEBAR effect was acknowledged there as a dominant mechanism causing  
28 slope currents.

29 One of the first efforts on modelling of baroclinic tides in the Malin  
30 shelf/slope area was undertaken by *Sherwin and Taylor* (1990) who used their  
31 two-dimensional slice model to reproduce the baroclinic tidal characteristics  
32 in the area. The model showed amplification of the baroclinic tidal signal  
33 near the bottom over the continental slope (presumably due to a baroclinic  
34 tidal beam), but their comparison with observations have been classified as  
35 inconclusive. Another attempt to build a sigma-coordinate numerical model  
36 suitable for investigation of internal tides generation in three dimensions was  
37 reported by *Xing and Davies* (1997). Their model was applied to the Ma-  
38 lin shelf-slope area, but since the runs were restricted to a two-dimensional  
39 topography the meridional variability of internal tides could not be inves-  
40 tigated. Note, however, that the most remarkable feature of the baroclinic  
41 tide there, viz. generation of an internal tidal beam over supercritical to-  
42 pography, was clearly demonstrated in (*Xing and Davies*, 1997, 1998). Some  
43 more details on the role of internal tide in enhancement of internal mixing  
44 and formation of the thermohaline structure of waters on the Malin Shelf  
45 were reported by *Xing et al.* (1999), but again, the three-dimensional effects  
46 of internal tides remained beyond the scope of this study.

47 Lack of understanding of the along slope variability of the dynamical  
48 processes developing in the area was a strong motivation for the 88-th cruise  
49 of RRS “James Cook” (hereafter JC88) to the Malin shelf-slope area con-  
50 ducted in June-July 2013 in the framework of the NERC funded project

FASTNet (UK). It was also assumed that the observational data set will be complemented by the results of three-dimensional modelling using the Massachusetts Institute of Technology general circulation model (hereafter MITgcm). This paper reports the results of the modelling efforts on replication of the processes developing in the area including baroclinic tides and bottom trapped internal waves. The model output is validated against the observational data collected during the JC88 cruise. The rest of the paper is organized as follows. A short description of the field campaign is presented in Section 2. Section 3 briefly outlines the model details. Section 4 discusses the generation mechanism of internal tides relevant to the present study. Section 5 presents the principal part of the modelling results with their comparison against collected observational data. Section 6 summarises all basic findings on bottom trapped internal waves. Some necessary theory to underpin formulated conclusion on theoretical and observational evidence of bottom trapped internal waves is presented in Section 7. The paper ends up with Summary and Conclusion presented in Section 8.

## 2. Observations

The principal objective of the JC88 cruise to the Malin Sea (28 June - 24 July, 2013) was investigation of cross-shelf transport and vertical mixing induced by internal tide and mesoscale dynamical processes developing in the area. In doing so 73 CTD stations were conducted on the shelf and over the continental slope. Each CTD cast was accompanied by velocity measurements using a 300 kHz Lowered Acoustic Current Profiler (LADCP) mounted on the CTD probe frame. The CTD survey was conducted along



three cross-sections shown in Figure 1. The positions of sampling stations are shown by green dots. Measurements along the middle and northern sections (sections 2 and 3, respectively) were conducted twice, with some stations repeated at the very same positions. Observational data from the stations depicted in Figure 1 by numbers are used for the present analysis; their details are presented in Table 1.

In addition to the stations, 14 moorings equipped with ADCPs were deployed in the area. The velocity time series recorded at four of them located over the continental slope, La, Lb, Sb, and Sd, are used in this study for the model validation. Two types of current meters were mounted at the moorings: 300 kHz ADCPs provided samplings at a rate 15 sec over a period of several weeks; sampling intervals of 75 kHz ADCPs varied from 2.5 to 60 min (depending on the device position). More details on the sampling intervals and moorings position are presented in Table 2.

### 3. Model set-up

The collected in-situ data were complemented by the results of numerical modelling conducted using the MITgcm. The central part of the model domain is shown in Figure 1 by a blue rectangle. It includes a  $536 \times 536$  model grid with horizontal resolution 150 m in zonal and meridional directions. In addition to the central part, 200 grid steps were added to each boundary of the model grid transforming it into a  $936 \times 936$  grid. In these extra lateral areas the grid resolution varied gradually from 150 m in its central part to 50 km near the boundaries. Such a telescopic increase of the horizontal steps towards the periphery makes physical size of the domain big enough to ex-

clude any reflections of the waves from the model boundaries for at least 10 tidal cycles. The vertical grid step was equal to 10 m.

The model was forced by barotropic tidal terms added to the right hand side of the momentum balance equations. The parameters of eight principal tidal harmonics

$$\begin{aligned} U_j &= A_j \cos(\omega_j t - \vartheta_j), \\ V_j &= B_j \cos(\omega_j t - \theta_j), \end{aligned}$$

( $j = 1, 2, \dots, 8$ ) are shown in Table 3. Zonal and meridional tidal discharges  $U_j$  and  $V_j$  with amplitudes  $A_j$  and  $B_j$  as well as tidal phases  $\vartheta_j$  and  $\theta_j$  for each tidal harmonic  $\omega_j$  were taken from the output of the inverse tidal model TPXO8.1 (*Egbert and Erofeeva, 2002*).

It worth mentioning that the principal target of the present study is tidally induced deep-water dynamics. All processes developing in the upper layer generated by ocean-atmosphere interaction (wind-driven circulation, inertial oscillations), or slope currents initiated by horizontal pressure gradients, as well as many other non-stationary processes of different nature were not an objective of the present study. Such an approach can be justified by an overwhelming predominance of tidal motion in the area, although some discrepancy between in-situ recorded and model predicted signals is expected.

After the MITgcm was configured, it was run in its barotropic mode (without vertical stratification of the fluid) to reproduce tidal currents; the latter were compared against the TPXO8.1 output for consistency. The forcing parameters were adjusted in such a way to achieve the best fit of both models.

122 After validation with the TPXO8.1, the MITgcm was run with realistic  
 123 fluid stratification. The data on the background temperature and salinity  
 124 distributions were taken from the CTD survey conducted during JC88 cruise.  
 125 A resulting vertical profile of the background buoyancy frequency averaged  
 126 over the whole set of the CTD stations is shown in the inset in Figure 1.

127 The vertical turbulent closure for the coefficients of vertical viscosity  $\nu$   
 128 and diffusivity  $\kappa$  was provided by the Richardson number dependent param-  
 129 eterization (*Pacanowski and Philander, 1981*):

$$\nu = \frac{\nu_0}{(1 + \alpha \text{Ri})^n} + \nu_b, \quad \kappa = \frac{\nu}{(1 + \alpha \text{Ri})} + \kappa_b.$$

130 Here  $\text{Ri} = N^2(z)/(u_z^2 + v_z^2)$  is the Richardson number, and  $N^2(z) = -g/\rho(\partial\rho/\partial z)$   
 131 is the buoyancy frequency ( $g$  is the acceleration due to gravity, and  $\rho$  is the  
 132 density),  $u$  and  $v$  are zonal and meridional components of horizontal veloc-  
 133 ities, respectively;  $\nu_b=10^{-5} \text{ m}^2 \text{ s}^{-1}$  and  $\kappa_b=10^{-5} \text{ m}^2 \text{ s}^{-1}$  are the background  
 134 parameters,  $\nu_0=1.5 \cdot 10^{-2} \text{ m}^2 \text{ s}^{-1}$ ,  $\alpha=5$  and  $n=1$  are the adjustable param-  
 135 eters. Such a parameterization increases coefficients  $\nu$  and  $\kappa$  in the areas  
 136 where the Richardson number is small which should take into account the  
 137 mixing processes induced by the shear instabilities and breaking internal  
 138 waves. The horizontal viscosity  $A^h$  and diffusivity  $K^h$  were taken at the level  
 139 of  $0.5 \text{ m}^2 \text{ s}^{-1}$ .

140 An extra block was added to the MITgcm that recorded velocities (from  
 141 surface to the bottom) with 1 min sampling interval at four grid points that  
 142 coincided with the positions of La, Lb, Sb, and Sd moorings (see Figure 1).

#### 143 4. Qualitative analysis of the generation mechanism

144 The regime of generation of internal waves by tide interacting with bottom  
 145 topography depends on its relative steepness. In a two dimensional case the  
 146 ratio of the maximum of the bottom steepness  $\gamma = dH/dx$  ( $H(x)$  is the water  
 147 depth) to the inclination  $\alpha$  of the characteristic lines of the wave equation,

$$\alpha(z) = dz/dx = \pm[(\omega^2 - f^2)/(N^2(z) - \omega^2)]^{1/2},$$

148 (here  $f$  is the Coriolis parameter) is a measure of the modal content of  
 149 the generated wave fields. The bottom is considered as a gently sloping one  
 150 when the inclination of characteristic lines  $\alpha$  substantially exceeds the bottom  
 151 inclination  $\gamma$ . In such a case the lowest baroclinic modes are generated with  
 152 the first mode being predominant.

153 In the areas with steep topographies where  $\gamma = \alpha$  (critical case) or  $\gamma > \alpha$   
 154 (supercritical conditions) a large number of baroclinic modes with compa-  
 155 rable amplitudes are generated. The superposition of the generated modes  
 156 leads to the formation of a tidal beam, i.e. a relatively narrow wave guides  
 157 along which the baroclinic tidal energy radiates from the bottom (*Vlasenko*  
 158 *et al.*, 2005). The baroclinic wave energy in tidal beams is concentrated along  
 159 the characteristic lines  $x = \pm \int dz/\alpha(z) + \text{const.}$

160 In our case considerable part of the Malin Slope below 500 m isobath  
 161 is supercritical for the semi-diurnal tide. This is clearly seen in Figure 2  
 162 where the characteristic lines are shown as pink stripes of different thickness  
 163 for three chosen sections depicted in Figure 1. The thickness of these lines  
 164 represents an intensity of tidal beams. In fact, the amount of tidal energy  
 165 converted from barotropic to baroclinic modes and concentrated inside the

166 beam depends on the size of the area of the bottom topography that satisfy  
 167 near-critical conditions, i.e.  $\gamma \sim \alpha$ .

168 Sharp edges in this respect (e.g. step-wise topography) produce sharper  
 169 but much weaker tidal beams than smoothly convex bumpy topographies. In  
 170 application to the considered area, sections 1 and 2 shown in Figure 2 with  
 171 near-critical conditions in wide sectors (a)-(b) and (c)-(d), respectively, can  
 172 produce much stronger tidal beam than narrow edge-type area (e)-(f) shown  
 173 in section 3.

174 This reasoning can be helpful in interpretation of observational data and  
 175 model results. Having known the positions of moorings and CTD stations  
 176 with respect to the tidal beam as show in Figure 2, one can expect a higher  
 177 level of tidal signal at depths where CTD station or mooring crosses the  
 178 beam.

## 179 **5. Comparison of observational and model data**

180 Comparative analysis of in-situ and model data is conducted separately  
 181 for sections 1, 2, and 3 shown in Figure 1.

### 182 Section 1.

183 Figure 3 represents model predicted zonal  $u$  and meridional  $v$  velocities  
 184 (left and central panels, respectively) calculated for the time  $t=85$  h of the  
 185 model run. As it is expected from the preliminary qualitative analysis, both  
 186 velocity sections clearly show evidence of a tidal beam. It crosses the verti-  
 187 cal line that depicts the position of station 6 at depths between 800 m and  
 188 1100 m. Both velocities reveal gradual change of the wave phase across the  
 189 beam:  $u$ -velocity changes from negative to positive, and  $v$ -velocity changes

190 its sign from positive to negative in the direction of wave phase propagation  
191 perpendicular to the beam (shown by the arrows  $C_p$ ).

192 Position of the tidal beam predicted by the model is consistent with the  
193 available observational data. The velocity profiles  $u$  and  $v$  recorded at CTD  
194 station 6 (Figure 3c) show the presence of local extremums in the water  
195 layer between 800 m and 1150 m (shaded yellow rectangle in the right panel)  
196 that can be associated with the beam. It is significant that the  $u$  and  $v$   
197 extremums have opposite signs, exactly as the MITgcm predicts for the layer  
198 with the tidal beam. Note, however, that the observational velocity  $v$  does  
199 not remain negative above 800 m depth as the model predicts, but changes  
200 its sign to positive which could be evidence of a slope current which is not  
201 included in the model.

202 Further evidence of the beam generation was found in the velocity time  
203 series recorded at moorings La and Lb. The moorings were deployed closer to  
204 the shelf break (closer with respect to CTD station 6, Figure 2), specifically,  
205 in the area between points **a** and **b**, Figure 2a, where the tidal beam is  
206 generated and is attached to the bottom. As a result, strong baroclinic tidal  
207 signal, produced by the beam, should be recorded in the bottom layer.

208 Figures 4a and 4c show 3.5-day time series of zonal and meridional veloc-  
209 ities recorded at moorings La and Lb. The model predicted velocities for the  
210 same positions are presented in Figures 4b and 4d. As it was expected from  
211 the qualitative analysis, strong intensification of tidal currents is evident in  
212 the bottom 600-800 m layer at mooring La, both in experimental and model  
213 data (panels a and b). This intensification is accompanied by the change of  
214 the wave phase in the direction from the bottom upward, Figures 4a and 4b,

215 which is a specific property of tidal beams.

216 Intensification of the velocity signals at the bottom are visible also in the  
217 model and experimental time series for the mooring Lb, although usage of  
218 two ADCPs (up-, and down-looking) with different sampling intervals (15 sec  
219 and 60 min, see Table 1) complicates the analysis and does not provide a  
220 smooth transition between two patterns (find white horizontal stripes at the  
221 depths 390-400 m in Figure 4 c) as it was for the mooring La. As a result, the  
222 theoretical and observational signals at the mooring Lb are less consistent.

## 223 Section 2.

224 Velocity time series recorded at mooring Sb and the model output for  
225 this location presented in Figure 5 a and 5 b do not reveal an overwhelming  
226 intensification of bottom currents associated with the tidal beam. Stronger  
227 bottom currents are still evident in  $v$ -component time series every second  
228 tidal cycle, Figure 5 a, but  $u$ -component is much weaker and does not show  
229 similar tendency. The model output reveals two fragments of strong bottom  
230 currents of  $v$ -velocity at  $t=0.8$  day and 1.8 day, Figure 5 b, but  $u$ -velocity has  
231 all its maxima at the free surface over the whole period. This result does not  
232 completely support the conclusions of the qualitative analysis which suggests  
233 similar tidal signal for both mooring, Lb and Sb (see Figures 2 a, and 2 b).

234 In this respect it worth mentioning here that the qualitative analysis of  
235 the generation mechanism presented schematically in Figure 2 is based on a  
236 two-dimensional (2D) theory that ignores any three-dimensional (3D) effects.  
237 As it was shown above, a 2D approach works perfectly well for section 1 where  
238 the topography is slightly curved, but nearly two-dimensional. As distinct  
239 from the moorings La and Lb, the mooring Sb was deployed in a canyon,

240 Figure 1, where 3D effects play a fundamental role in formation of tidal wave  
 241 fields. They can lead either to focussing, or defocussing of tidal energy. The  
 242 latter probably was the case with the mooring Sb that does not reveal strong  
 243 bottom intensification of tidal currents.

244 It is interesting that the model output also does not show strong bottom  
 245 currents in the canyon. The reason for this effect can be analysed separately  
 246 (normally canyons are traps for tidal energy, *Vlasenko et al. (2016)*), but the  
 247 consistency of the model output with in-situ data (except the fact that the  
 248 ADCP time series are more noisy due to high frequency processes that are  
 249 not captured by the model) looks promising for the current study showing  
 250 that the model captures all main features of tidal cycle correctly.

251 3D effects develop locally in canyons and attenuate in the far field and  
 252 can not suppress completely the mechanism of tidal beam generation. As  
 253 a confirmation of that, the velocity profiles recorded at CTD stations 37  
 254 and 45, conducted offshore of the canyon on 1400 m isobath, clearly show  
 255 evidence of the tidal beam descending in the deep water, Figure 5 c and 5 d.

256 The tidal origin of these strong deep water currents can be confirmed by a  
 257 simple comparison analysis. The time difference between velocity samples at  
 258 stations 37 and 45 was 5 days 6 hours and 56 min (see Table 2). In terms of  
 259 tidal activity it comprises 10.25 and 10.6 periods for two tidal harmonics,  $M_2$   
 260 and  $S_2$ , respectively, or nearly 0.5 tidal cycle time lag on average. As one can  
 261 see in Figures 5 c and 5 d, below 1000 m depth the velocity  $u$  is positive for  
 262 station 37 and negative for station 45, and velocity  $v$  is negative for station  
 263 37 and positive for station 45, which should be valid for periodic processes  
 264 when considering a half-period time lag.



265 Section 3.

266 Strong tidal beam signal at section 3 was not expected from the quali-  
267 tative analysis, Figure 2. As a confirmation of that, the beam is not clearly  
268 observed in the model output and in the ADCP time series recorded at  
269 mooring Sd, Figure 6 a and 6 b. Overall, horizontal velocities observed at the  
270 mooring Sd are quite weak and polluted by a high frequency noise that is not  
271 present in the model signal. There are still two fragments with maximums  
272 of  $v$  velocity near the bottom in the middle of the theoretical time series,  
273 but general tendency is that the in-situ data and the model output reveal a  
274 dominantly barotropic tidal signal.

275 Apart from sharp edge topography discussed in previous section, another  
276 reason for a weak tidal energy conversion in section 3, could be the weak  
277 stratification in the layer of tidal beam generation. The inset into Figure 1  
278 shows that the buoyancy frequency at the depths of 200-250 m (the place of  
279 the beam generation) has a local minimum.

280 Note, however, that in the abyssal part of section 3 the beam is quite  
281 visible. As evidence of that, the instant vertical profiles of horizontal ve-  
282 locities recorded at CTD station 14 (located seaward from Sd mooring, see  
283 Figures 1) do not reveal any notable activity in the layer between 400 m and  
284 500 m depth, but demonstrate an overwhelming signal below 700 m with lo-  
285 cal maximums of both components at the depth of about 800 m, Figure 6 c.  
286 A realistic explanation of the absence of the tidal beam signal at mooring  
287 Sd and its presence in the deep part, could be an assumption that the tidal  
288 beam in this section is generated deeper than the 500 m isobath.

## 289 6. 3D deep water dynamics

290 The prior cross-section analysis has shown that although the 2D theory is  
291 a good tool for a qualitative analysis, it is not always applicable to realistic  
292 3D problems. As demonstrated, the tidal characteristics in all three sections  
293 is quite different, and the main reason for that could be the substantial  
294 three-dimensionality of the bottom topography. It is postulates here that the  
295 bottom promontory depicted in Figure 7 by a green rectangular box, could  
296 be the reason for observed along-slope variability of the baroclinic tidal fields.

297 This variability is clearly seen in the deep water below 700 m depth. Fig-  
298 ure 8 shows evolution of the temperature field in the vertical sections depicted  
299 in Figure 7 by red line. The temperature fields in this time series, Figure  
300 8, were taken with 2-hour time interval. It show an internal wave structure  
301 that propagates in the north-east direction from the promontory. The crest  
302 of this wave is marked by an ellipse. An estimate of the wave phase speed  
303 based on Figure 8 gives the value  $0.57 \text{ ms}^{-1}$ .

304 Another estimates of the wave parameters can be done using Figure 9  
305 which shows a plan view of a spatial distribution of the temperature for the  
306 northward propagating wave system at the depth of 1000 m. Temperature  
307 fields in this figure are given for the same time periods as in Figure 8. It is  
308 clear from Figure 9 that the wave system is attached to the slope. Its intensity  
309 decreases seaward revealing properties of a bottom trapped internal wave.

310 The wavelength can be estimated from the distance between the positions  
311 of the wave crest and the wave trough. The latter can be associated with the  
312 local minimum and maximum of the water temperature, respectively. Most  
313 clearly they are visible at the time moment 77 h. As it is shown in this panel,

314 the distance  $\lambda/2$  between the trough and crest is equal to 13.9 km. Assuming  
 315 a semidiurnal tidal periodicity of the wave process one can estimate the phase  
 316 speed as  $0.64 \text{ ms}^{-1}$ . Similar values calculated using just the positions of  
 317 the trough and crest at different moment of time (the distance between the  
 318 positions **a** and **b**, as well as **c** and **d**, see Figure 9) are  $0.71 \text{ ms}^{-1}$  and  $0.61$   
 319  $\text{ms}^{-1}$ . Thus, all estimates give quite consistent values of the phase speed in  
 320 the range between  $0.57 \text{ ms}^{-1}$  and  $0.71 \text{ ms}^{-1}$ . The average from all is equal  
 321 to  $0.62 \text{ ms}^{-1}$ , which defines the wavelength as 26.8 km.

## 322 **7. Analytical solution for bottom trapped internal waves**

323 Numerical solution presented in Figures 8 and 9 are characteristic of a  
 324 bottom trapped internal wave. The wave is confined to the bottom and exists  
 325 only below 800 m depth; it propagates northward leaving the shelf to its right  
 326 and its amplitude attenuates in seaward direction. To be confident that this  
 327 interpretation is correct let's compare characteristics of the model predicted  
 328 wave against analytical solution developed by *Huthnance* (1978). Here we  
 329 restrict our discussion to the problem formulation and the final solution,  
 330 assuming that more details can be found in the original paper.

331 A rotating basin of variable depth filled with stratified water at the lat-  
 332 itude of  $56^\circ$  is considered. The horizontal axis  $0x$  and  $0y$  of the Carte-  
 333 sian coordinate system are taken seaward and along coastline, respectively.  
 334 The vertical axis is oriented upward so that the water depth is defined as  
 335  $z = -H(x)$ , with uniform bottom topography in  $y$  direction. The coastline  
 336 is set at  $x = 0$ . The bottom profiles  $H(x)$  was taken by averaging of several  
 337 zonal topography cross-sections in the area near the latitude of  $56^\circ$ , Figure

338 7 (sections a, b, c).

339 We are seeking for a periodic wave solution  $\sim \exp[i(ky + \omega t)]$  with the  
 340 wave-number  $k$  and frequency  $\omega$  of the Boussinesq type equations for strat-  
 341 ified incompressible fluid. For internal waves propagated along topography  
 342 the governing system can be reduced to one equation for the pressure ampli-  
 343 tude  $P(x, z)$ , i.e. the pressure  $p(x, y, z, t) = P(x, z) \exp[i(ky + \omega t)]$ :

$$\frac{\partial^2 P}{\partial x^2} + \frac{(1 - \sigma^2)}{S} \frac{\partial}{\partial z} \left( \frac{1}{N^2} \frac{\partial P}{\partial z} \right) - l^2 P = 0, \quad (1)$$

344 with the following boundary conditions:

$$\begin{aligned} \frac{dH}{dx} \left( \frac{\partial P}{\partial x} + \frac{l}{\sigma} P \right) + \frac{1 - \sigma^2}{SN^2} \frac{\partial P}{\partial z} &= 0 \quad \text{at } z = -H(x), \\ \frac{\partial P}{\partial z} &= 0 \quad \text{at } z = 0, \\ P &\rightarrow 0, \quad \text{at } x \rightarrow \infty, \\ kP - \sigma \frac{\partial P}{\partial x} &= 0 \quad \text{at } x = 0. \end{aligned} \quad (2)$$

345 Here  $l = kL$  and  $\sigma = \omega/f$ , are a non-dimensional wave number, and non-  
 346 dimensional frequency, respectively.

347 A wide variety of oceanographic situations are discussed in (*Huthnance*,  
 348 1978), but we focus here on just one particular case that fits the Malin Sea  
 349 slope conditions. A non-dimensional parameter that shows relative contribu-  
 350 tion of rotation and stratification to the wave process is the Burger number,  
 351  $S = (N_{\max} H / f L)^2$ , where  $H_0$  and  $L$  are the scales for the depth and slope  
 352 width, respectively,  $N_{\max}$  is the maximum of the buoyancy frequency, and  $f$   
 353 is the Coriolis parameter. Bearing in mind that the considered wave process  
 354 develops below 800 m depth, the maximum of the buoyancy frequency can  
 355 be taken  $N_{\max} = 3 \cdot 10^{-3} \text{ s}^{-1}$  (see Figure 1). For two other parameters we

356 can take:  $L = 40000$  m and  $H_0 = 2000$  m. Then the Burger number can be  
 357 estimated as  $S \cong 1.2$ .

358 An analytical solution for the bottom-trapped internal waves with the  
 359 wave number  $k$  was obtained by *Huthnance* (1978) for the conditions when  
 360  $1/(kL) < S^{-1/2} < 1$ . As it was found in the previous section, the wavelength  
 361 of the model predicted wave can be estimated as 26.8 km, i.e. parameter  
 362  $1/(kL) \cong 0.11$ . For the Burger number  $S = 1.2$  the value  $S^{-1/2} = 0.91$ , i.e.  
 363 theory by *Huthnance* (1978) can be applied to the Malin Seas slope area.

364 For relatively short-scale topographically trapped internal waves with the  
 365 wavelength in the range between 10 km and 30 km, the solution of (1)-(2)  
 366 reads:

$$P(x, z) = H_m(\beta(x)) \exp \left[ -\frac{\beta^2(x)}{2} \right] \exp \left[ -k \frac{z + H(x)}{\sqrt{\phi(z) + \left( \frac{dH}{dx} \right)^2}} \right], \quad (3)$$

367 where  $H_m$  is the Hermite function of the  $m$ -th order,

$$\beta(x) = (x - x_0) \frac{N_{\max} H_0}{f} \left[ \frac{\left( \frac{dH}{dx} \right)^2 \frac{d^2 \phi}{dz^2}}{2\phi(z)} - \frac{\frac{d^3 H}{dx^3}}{\frac{dH}{dx}} \right]_{(x_0, z_0)},$$

368

$$\phi(z) = \frac{(f^2 - \omega^2)}{N^2(z)} \left( \frac{L}{H_0} \right)^2.$$

369 Here  $(x_0, z_0)$  are the coordinates of the sea bed point where the product  
 370  $N(z) \frac{dH(x)}{dx}$  has maximum value. For the bottom profile under consideration  
 371 here the maximum of buoyancy frequency (or main pycnocline) is the depth  
 372 where the core of the bottom trapped wave should be expected.

Figure 10 presents first three normalized amplitude functions  $P(x, z)$  of solution (3). The calculations were done for the frequency  $\omega$  that corresponds to  $M_2$  tide and the wavelength 26.8 km. For node  $m = 0$  the solution has one maximum located at the bottom with its centre at the isobath of 1200 m. Centres of node  $m = 1$  and mode  $m = 2$  solutions are located nearly at the same depth, but they have two and three maximums, respectively.

In order to understand which of the nodes was generated in the numerical model over the Malin Sea slope one can analyse the position of amplitude of the propagated wave. In doing so, the field of the amplitude velocities

$$a = \sum_{j=1}^{12} \sqrt{u^2(j) + v^2(j)} / 12;$$

along three vertical sections shown in Figure 7 has been calculated. Here  $u(j)$  and  $v(j)$  are zonal and meridional model predicted amplitude velocities taken over one tidal period.

Figure 11 shows the  $a$  field for three vertical sections presented in Figure 7 by blue dotted lines. Apart from a totally expected intensification of barotropic tidal currents in the shallow water area to the right of the longitude  $9.4^\circ\text{W}$ , two localised spots of intense baroclinic tidal motions can be identified in Figure 11 c. One of these areas can be classified as a tidal beam traced in this panel by a characteristic line of the wave equation depicted by solid black line. The red spot attached to the bottom between  $9.45^\circ\text{W}$  and  $9.65^\circ\text{W}$  (marked by a magenta ellipse) has nothing to do with the tidal beam. It exists separately attached to the bottom where trapped internal wave propagates.

In the middle section (Figure 11 b) these two objects still exist separately

below 800 m, but above this depth they merge. In the northernmost section  
a (Figure 7) the bottom trapped wave is less intense (see Figure 11 a), but  
it is still visible as bottom intensified current. Assuming that Figure 11  
shows evidence of the bottom trapped internal wave, one can conclude from  
its compact structure that it represents node  $m = 0$  of solution (3) of the  
boundary value problem (1)-(2). Comparing Figures 11 with upper panel of  
Figure 10 one can find quite a good consistency, both in spatial structure,  
and in the depth of its location, specifically in the area where  $N(z)H'(x)$   
takes the maximum value.

## 8. Discussion and conclusions

The properties of bottom trapped waves can be treated in terms of theory  
developed by *Wang and Mooers* (1976). According to their classification  
internal waves in stratified water can exist with scales of internal Rossby  
radius that propagate in a basin of nearly constant depth. On the other  
hand, in homogeneous ocean with an inclined bottom the topographic shelf  
waves can exist. In real ocean water stratification and inclined bottom exist  
together. This can lead to the generation of a hybrid type waves, so called  
bottom trapped waves, that have properties of both. The frequency of Rossby  
wave is always below the inertial frequency  $f = 2\Omega \cos \phi$  ( $\Omega = 0.0000729 \text{ s}^{-1}$   
is angular velocity of Earth rotation and  $\phi$  is the latitude). However, the zero  
mode internal Kelvin wave can exist both below and above critical latitude,  
having  $M_2$  tidal period, in particular.

The most typical mechanism for generation of bottom trapped internal  
waves is scattering of internal wave propagating along corrugated topography.

420 Interacting with a prominence a superinertial internal wave ( $\omega > f$ ) trans-  
 421 forms its energy to a system of scattered internal waves freely radiated in all  
 422 directions from the topography feature (see *Chapman* (1982), for instance).  
 423 However, as it was shown by *Huthnance* (1978); *Dale and Sherwin* (1996);  
 424 *Dale et al.* (2001), if the wave frequency is close to the inertial one (as in our  
 425 case,  $\sigma/f = 1.178$ ), the wave is “nearly” trapped by the topography. In other  
 426 words, the energy of the incident wave is split into two parts, one goes to  
 427 internal Poincare modes radiated from the source of generation, and another  
 428 one to the topography trapped internal wave propagating along the coast  
 429 leaving the coastline to its right (in the northern hemisphere). Moving closer  
 430 to the inertial frequency, less energy is transformed to the radiated Poincare  
 431 modes. The vertical structure of these waves depends both on slope/shelf  
 432 topography and fluid stratification.

433 All these theoretical issues are highly relevant for explanation of bottom  
 434 intensification of tidally induced baroclinic currents observed in the 88-th  
 435 cruise of the RRS ”James Cook” over the Malin Sea continental slope and  
 436 reproduced numerically using the MITgcm. The reported data has quite  
 437 a realistic explanation in terms of superposition of two energetic processes  
 438 developing in the area. One of them is a bottom trapped internal wave  
 439 generated by the tidal flow at a local bottom prominence and propagated  
 440 along the slope (Figures 7-9). The maximum currents associated with this  
 441 type of motions were located near the bottom at the depths below 800 m.  
 442 The intensity of the currents decreased from its centre both upward and  
 443 horizontally from the core. The analysis of the position of the bottom current  
 444 and its spatial distribution allows one to conclude that recorded in-situ and



445 predicted numerically baroclinic processes with good accuracy can be treated  
446 in terms of analytical solution (3) presenting a zero-node bottom trapped  
447 internal wave (its spatial structure is shown in Figure 10 a).

448 The second class of motions identified over the Malin Slope is a baroclinic  
449 tidal beam that is usual feature of internal tidal fields in the area of steep  
450 (supercritical) topographies. The beam is normally generated at the shelf  
451 break and radiates tidal energy into abyss along the characteristic lines of the  
452 internal wave equation. Evidence was found for this in both the observational  
453 data set and in the model output (Figure 11, for instance).

454 It is worth noting that the properties of the tidal beam were discussed  
455 here only for the three cross-slope sections where the observational data  
456 were available. However, assuming the more complicated three-dimensional  
457 dynamics, one can learn more from the model output on the spatial char-  
458 acteristics of the tidal beam. Figure 12 shows that the tidal beam can be  
459 detected in much wider area than that occupied by the bottom trapped inter-  
460 nal wave. The tidal beam is not visible in the temperature fields in Figure 9  
461 which allowed identification of the bottom trapped wave. However, the more  
462 energetic horizontal motions presented in Figure 12 illustrate the seaward  
463 propagation of the tidal beam phase over the whole area during one tidal  
464 cycle quite clearly. The along slope stretched blue area of negative eastward  
465 velocity  $u$  visible at  $t=73$  h migrates seaward over next 6 h being gradually  
466 replaced near the slope by the red colour (positive velocity) stripe that also  
467 moves seaward. A distinguishing feature of the tidal beam, i.e. downward  
468 radiation of baroclinic tidal energy, can also be identified in Figure 12. The  
469 left-top corner of each panel in this figure do not reveal strong variation of

470 horizontal velocity suggesting that the tidal beam in this area is located much  
471 deeper.

472 There is also a possibility to record the waves arrived to the Malin Shelf  
473 from some distant sources of generation, the Rockall Bank or the Anton  
474 Dohrn Seamount, for instance. Such events were observed from space (*Small*  
475 *et al.*, 1999b) and analysed theoretically (*Stashchuk and Vlasenko*, 2005).  
476 Note, however, that in light of the present study which focuses on the bottom  
477 trapped internal waves, such a scenario is considered as less probable than  
478 local generation, although some further work would be necessary to address  
479 this issue.

480 Overall, it is not always easy to separate with certainty the along slope  
481 propagated bottom trapped waves from downward radiated tidal beams. An  
482 obvious criteria for such spatial separation would be the fact that bottom  
483 trapped internal waves always have their maximum velocities at the bottom,  
484 Figure 10, whereas the beams always detach from the place of generation  
485 having maximums of their velocities in the intermediate layers, Figure 11.  
486 However, in the areas where the beam is generated (shallower 1000 m isobath)  
487 these two processes develop together and contribute equally to the bottom  
488 current intensification. As an illustration of that, Figure 13 shows a spa-  
489 tial distribution of the amplitude of horizontal velocity in the bottom layer.  
490 Only joint analysis with Figure 11 allows us to conclude that the bottom  
491 trapped internal waves are responsible for intensification of bottom currents  
492 deeper 1000 m isobath (find red spot in the top right corner of Figure 13) but  
493 shallower this depth mostly the tidal beam contributes to bottom currents.

494 Analysis conducted here illustrates evidence of three-dimensional baro-

495 clinic wave dynamics (such as bottom trapped internal waves and 3D tidal  
496 beams) developing over the Malin Sea continental slope. Importantly, con-  
497 sidered here configuration of bottom topography is not unique to this study  
498 and is expected to have global significance.

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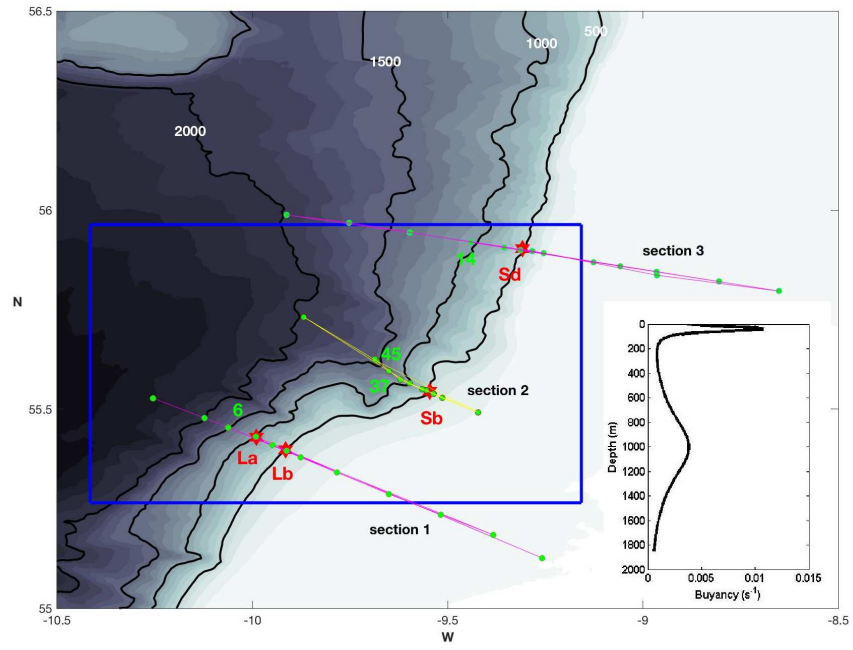


Figure 1: Bathymetry of the Malin Sea with the plan view of the JC88 field experiment. Green closed circles show positions of the CTD stations; red hexagrams depict the positions of moorings. Blue rectangle shows the model domain. Buoyancy frequency averaged over CTD stations is shown in the inset.



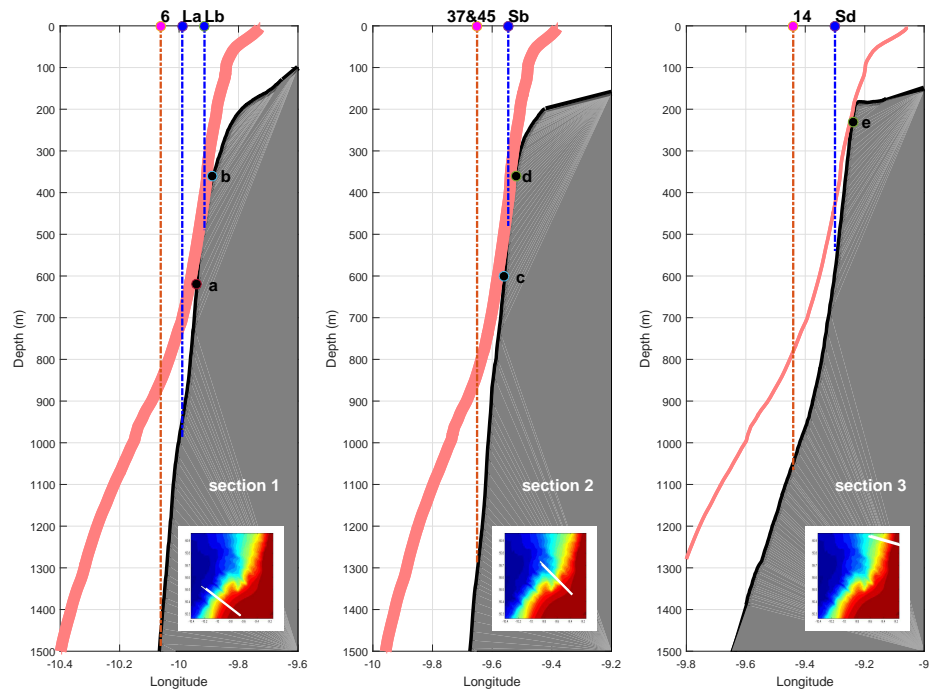


Figure 2: Bottom topography along sections 1, 2, and 3 (from the left to the right) depicted in Figure 1. Location of moorings Lb, Sb, Sd, and CTD stations 6, 37, 45, & 14 are shown by vertical dash-dotted lines. Red lines show positions of a tidal beam.

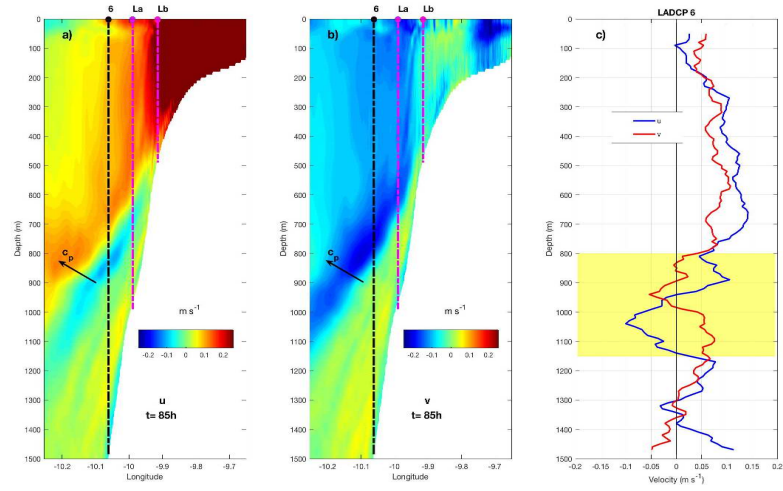


Figure 3: Section 1 (see Figure 1): Model predicted zonal  $u$  (a) and meridional  $v$  (b) horizontal velocities for the time moment  $t=85\text{h}$  of the model run. The characteristic line is shown by the black dotted line. (c) Instant vertical profiles of  $u$  and  $v$  velocities recorded by LADCP at CTD station 6.

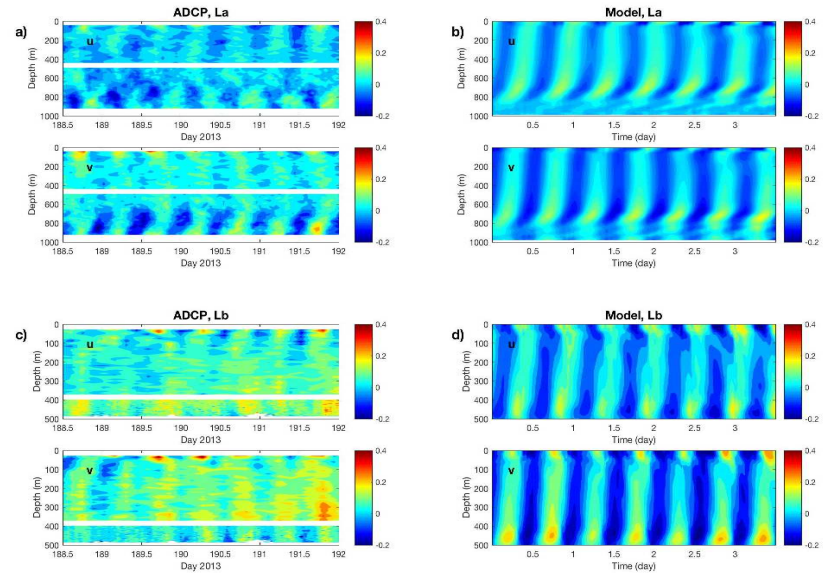


Figure 4: Zonal  $u$  and meridional  $v$  velocity time series recorded at moorings La (panel a) and Lb moorings (panels c). Corresponding model predicted time series are shown in panel (b) and (d), respectively.

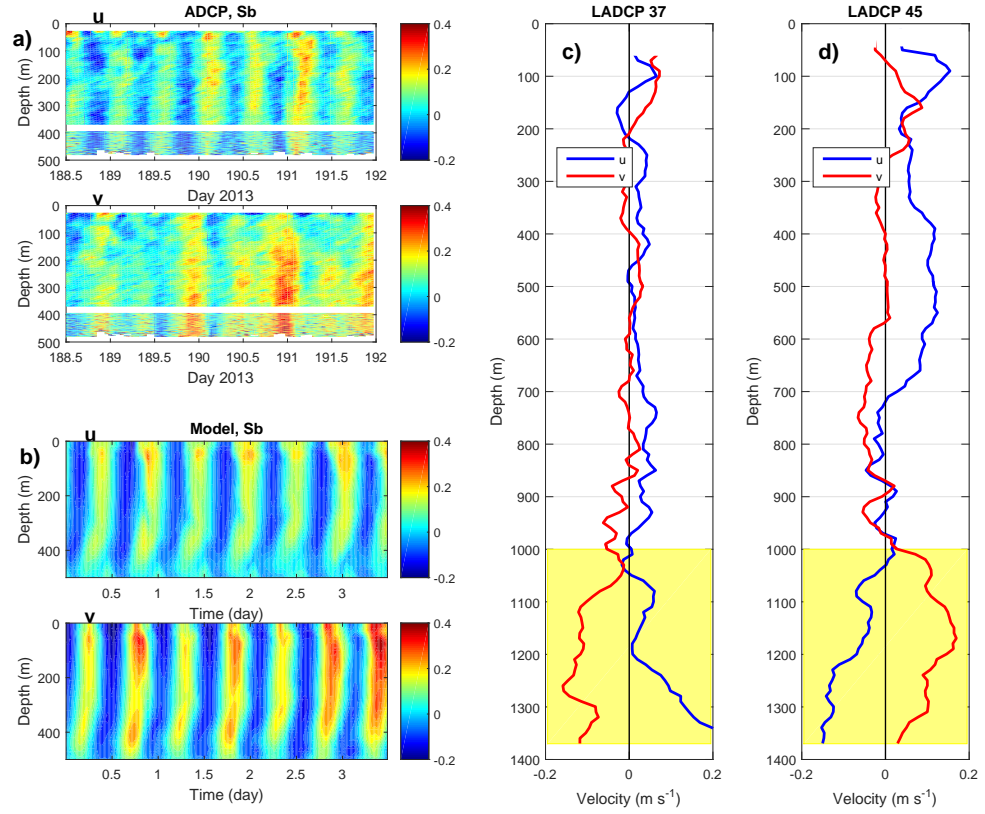


Figure 5: Zonal  $u$  and meridional  $v$  velocity time series recorded at Sb mooring by a) ADCP and b) the model. c) Instant  $u$  and  $v$  velocity profiles measured at stations 37 and 45.

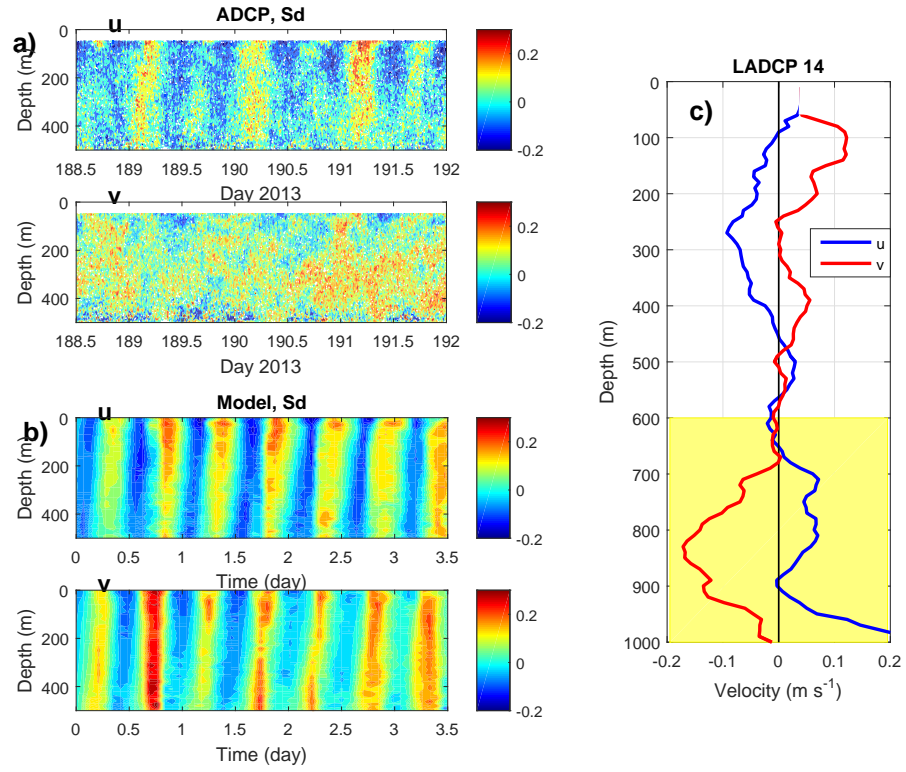


Figure 6: Zonal  $u$  and meridional  $v$  velocity time series recorded at Sd mooring by a) ADCP and b) the model. c) Instant  $u$  and  $v$  velocity profiles measured at CTD station 14.

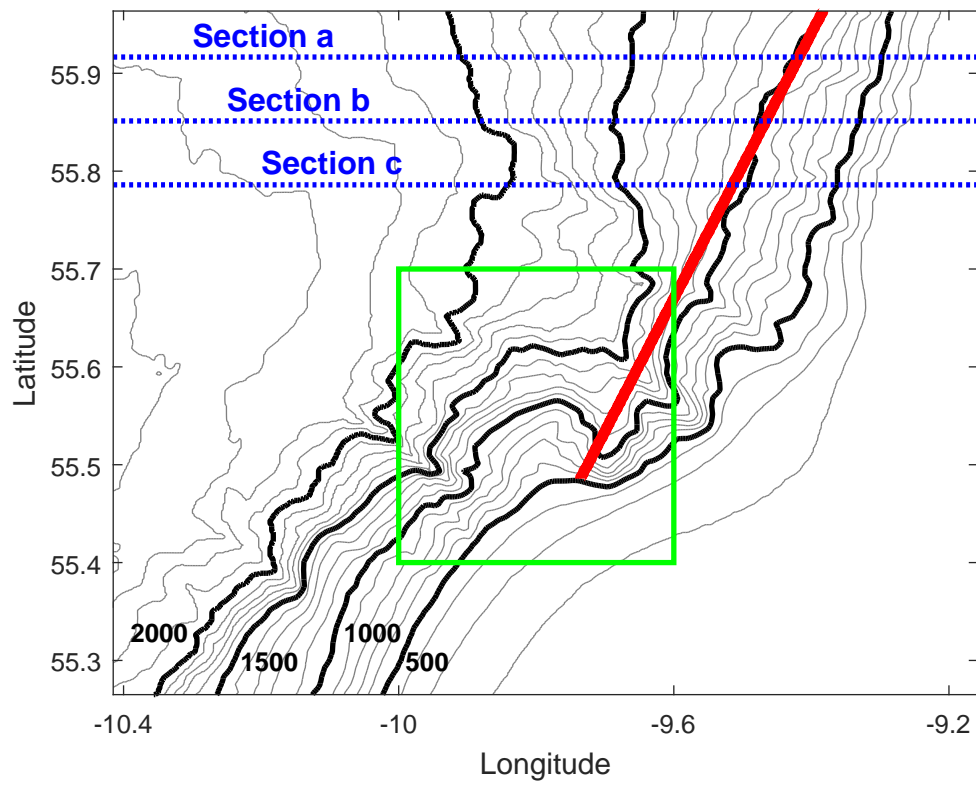


Figure 7: Bottom topography with sections used in the analysis.

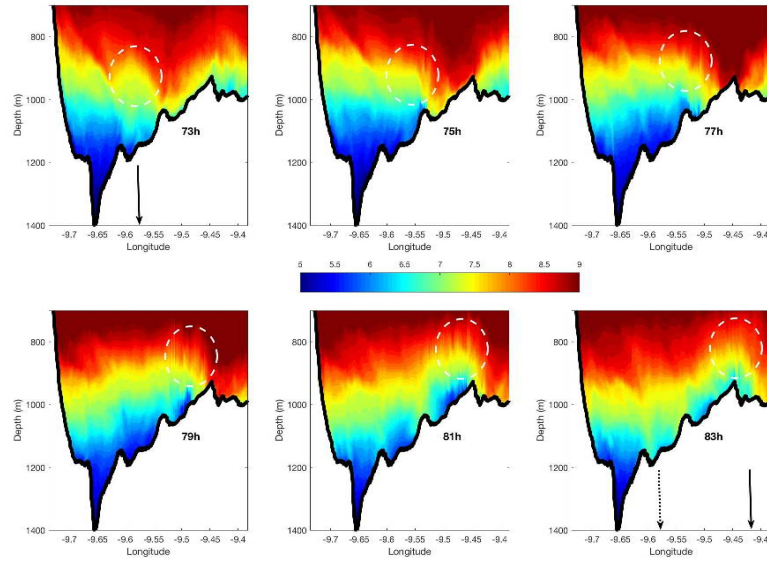


Figure 8: Model predicted evolution of the temperature field along the vertical section shown in Figure 7 by red line. The time span covers interval from  $t=73$  h to  $t=83$  h of the model time. White dashed ellipses depict the positions of the wave crest.

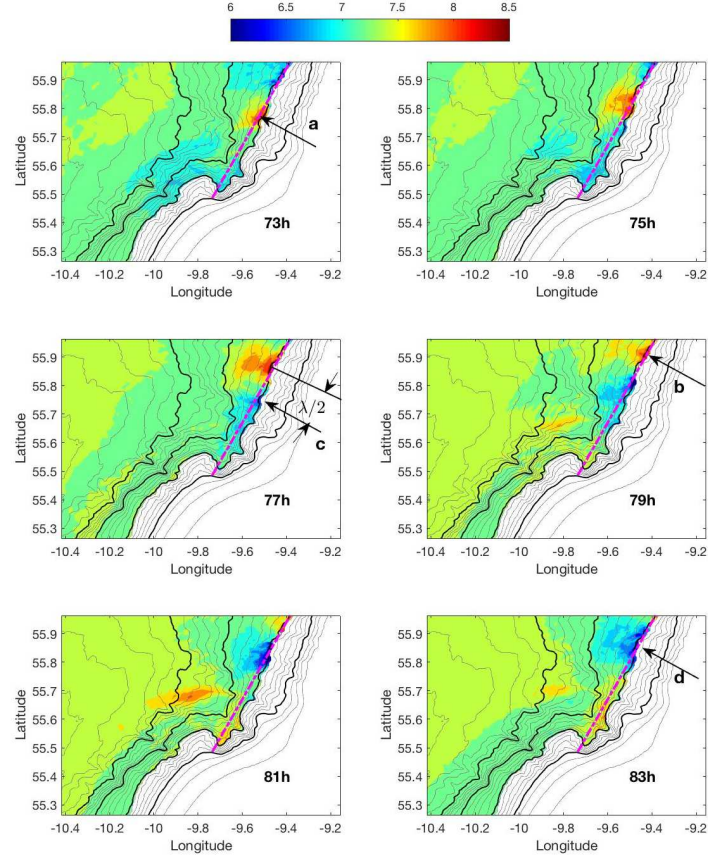


Figure 9: Model predicted temperature at 1000m depth. Model time is shown in right bottom corner of each panel. Black arrows show positions of troughs (red spots) and crests (blue colour) of the bottom trapped internal wave.



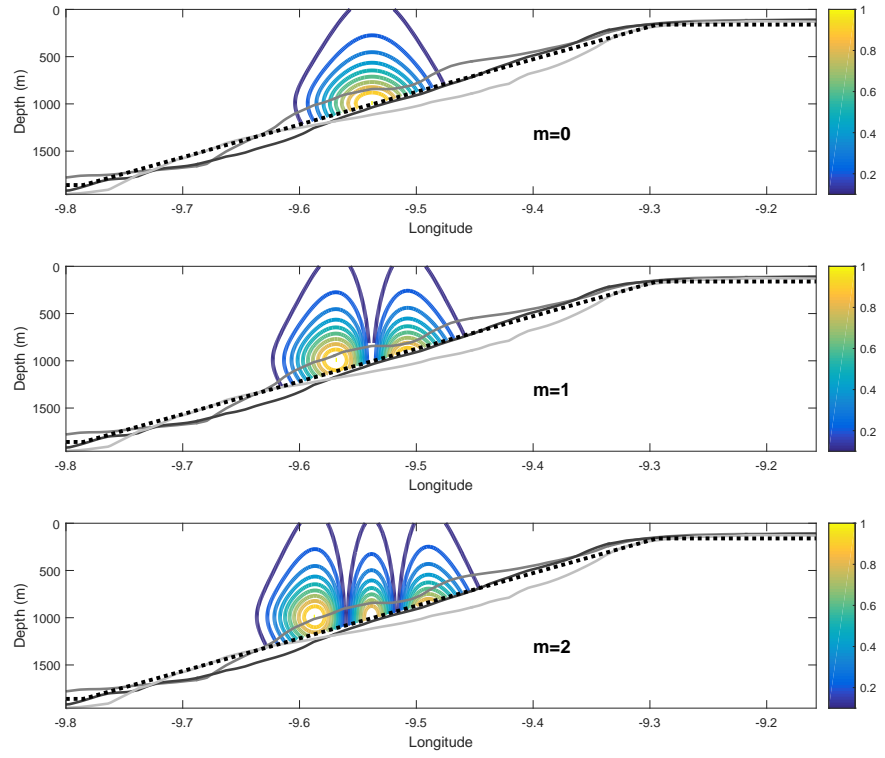


Figure 10: Three first normalized eigen functions  $P(x, z)$  of the boundary value problem (1)-(2). Dotted line shows the bottom topography calculated as an average from several profiles presented in the figure.

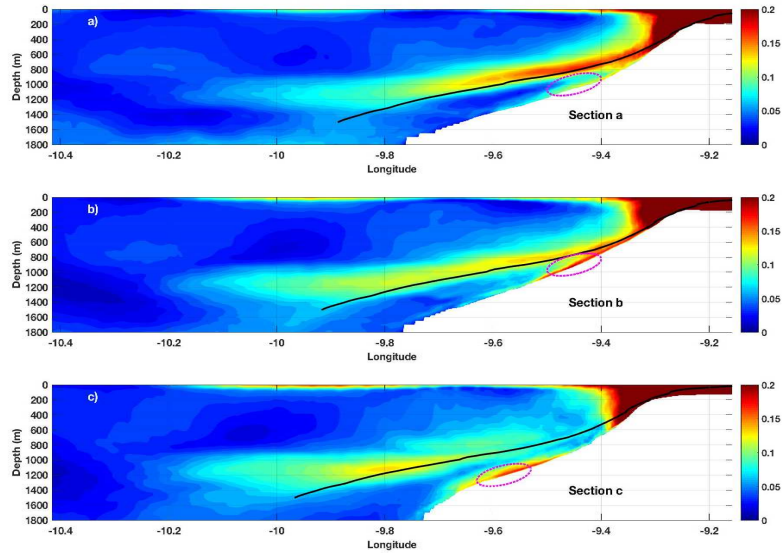


Figure 11: Model predicted amplitude of horizontal velocities for zonal sections a)  $55.9166^\circ$ , b)  $55.8513^\circ$  and c)  $55.786^\circ$  shown in Figure 7. Positions of the bottom trapped internal wave are shown by the magenta ellipses. The direction of the tidal beam is depicted by the black lines.

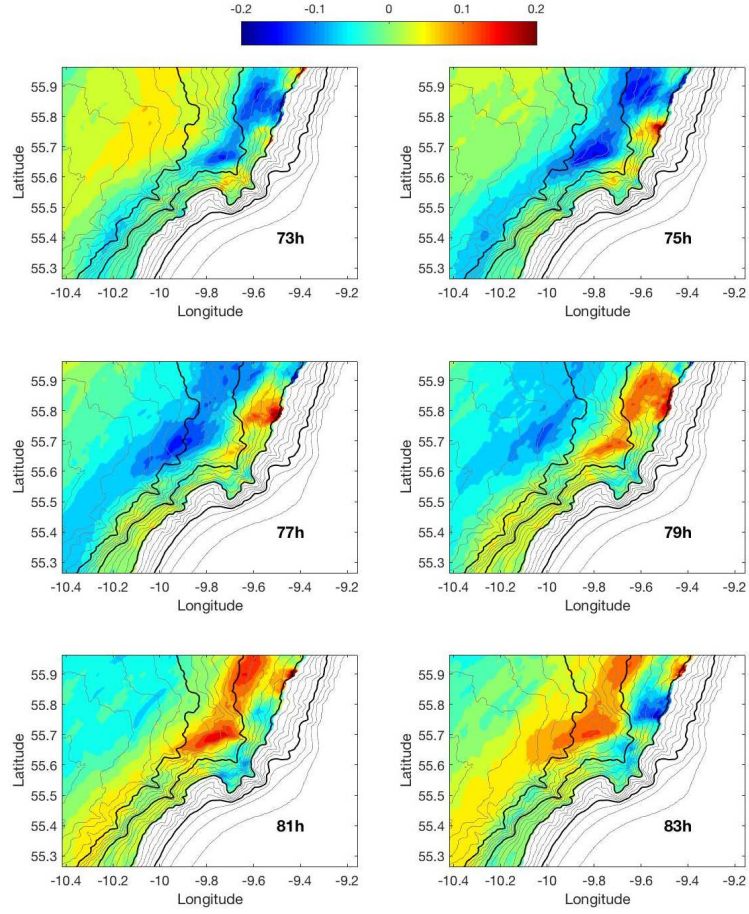


Figure 12: Model predicted  $u$ -velocity at 1000m depth. Time span is shown in right bottom corner of each panel.

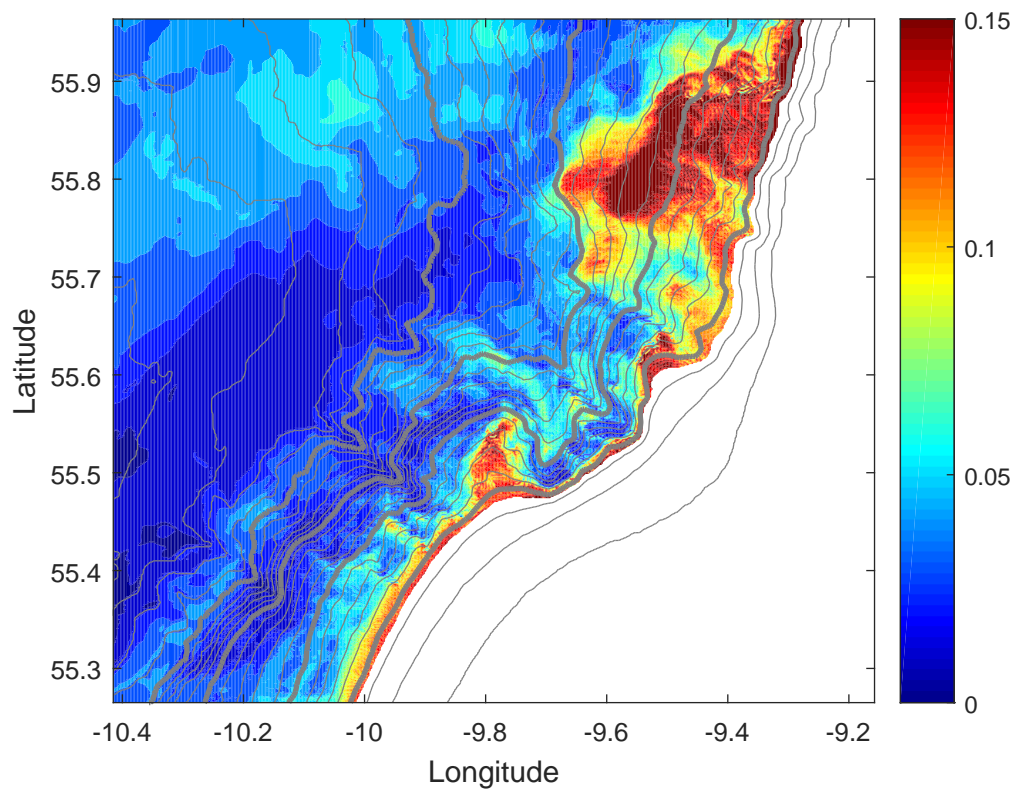


Figure 13: Model predicted amplitude of horizontal velocity ( $\text{ms}^{-1}$ ) 10 m above the bottom overlaid with bottom topography.

Table 1: CTD stations details

Name	Latitude	Longitude	Date	Time	Depth (m)
06	55°27.23'	010°03.69'	01/07/2013	07:00	1483
37	55°35.761'	009°39.037'	06/07/2013	22:36	1380
45	55°35.760'	009°39.041'	12/07/2013	05:31	1384
14	55°55.229'	009°26.42'	02/07/2013	22:32	1044

Table 2: ADCP mooring data

Name	Instrument	Up/Down	Depth (m)	Sample interval
La	ADCP 75KHz	U	467.5	60 min
	ADCP 75KHz	D	472	60 min
Lb	ADCP 75KHz	U	391	60 min
	ADCP 300KHz	D	394.5	15 sec
Sb	ADCP 75KHz	U	387.5	5 min
	ADCP 300KHz	D	389	15 sec
Sd	ADCP 75KHz	U	544	2.5 min

Table 3: Tidal data

	M2	S2	N2	K2	K1	O1	P1	Q1
$A(\text{m}^2\text{s}^{-1})$	30	14	8.2	3.5	1.78	0.98	0.67	0.56
$\vartheta(\text{degr})$	143	127	75	102	170	0	39	18
$B(\text{m}^2\text{s}^{-1})$	25	9.14	7.18	2.79	3.11	1.32	1.09	0.9
$\theta(\text{degr})$	28	180	125	146	115	63	78.5	47.5